# Analysis of Flow from a Dewar

7-23-07

## *Vapor Flow*

Consider the general case where there is a flow of saturated vapor out of the dewar (see figure 1). The vapor flows through a tube of some kind with a mass flow rate of $\dot{m}_{out}$, and exits at a pressure of $p_a$. The pressure inside the dewar is $p_{in}$, and the total mass of nitrogen in the dewar is $M$. The total rate of heat flow into the dewar (whether from unintentional parasitics or other means) is given by $q$. The total volume of the dewar is constant and assumed to be known and equal to $V$. For this analysis, kinetic and potential energies will be neglected.

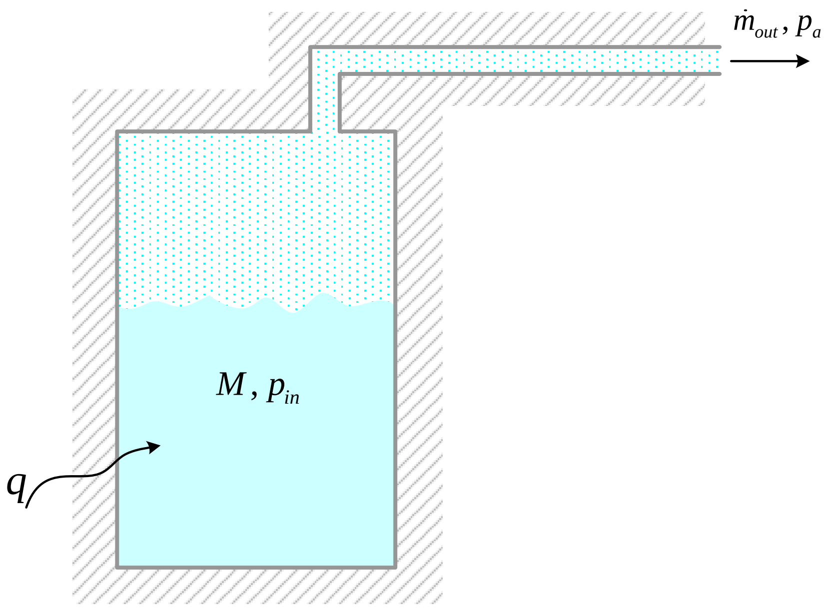


*Figure 1. Diagram of saturated vapor flow from a dewar*

## General Analysis

Conservation of mass:

$$\dot{M} = -\dot{m}_{out} \tag{1}$$

Knowing the geometry of the outlet pipe and properties of the saturated vapor, the mass flow rate can be related to the pressure drop across the pipe:

$$\dot{m}_{out} = k\left(p_{in} - p_{out}\right) \tag{2}$$

Conservation of volume (i.e. the dewar's volume is known and constant):

$$\begin{aligned} V &= M\left(xv_g + (1-x)v_f\right) \\ &= M\left(xv_{fg} + v_f\right) \end{aligned} \tag{3}$$

Conservation of energy at dewar boundary:

$$q = \frac{dE}{dt} + \dot{m}_{out} h_g$$
$$= \frac{d}{dt}\left[ M\left( x u_{fg} + u_f \right)\right] + \dot{m}_{out} h_g \qquad (4)$$
$$= \dot{M}\left( x u_{fg} + u_f \right) + M\left( \dot{x} u_{fg} + x \dot{u}_{fg} + \dot{u}_f \right) + \dot{m}_{out} h_g$$

Note that $h_g$ is only a function of $p_{in}$. Also note that $u_{fg}$ and $u_f$ are both only functions of $p_{in}$. Using the chain rule:

$$q = \dot{M}\left( x u_{fg} + u_f \right) + M\left( \dot{x} u_{fg} + x \frac{du_{fg}}{dp_{in}} \dot{p}_{in} + \frac{du_f}{dp_{in}} \dot{p}_{in} \right) + \dot{m}_{out} h_g \qquad (5)$$

Conservation of energy at pipe boundary:

$$\dot{m}_{out} h_g = \dot{m}_{out} h_{out} \;\Rightarrow\; h_{out} = h_g \qquad (6)$$

Plugging (1) and (2) into (5):

$$q = -k\left( p_{in} - p_{out} \right)\left( x u_{fg} + u_f \right) + M\left( \dot{x} u_{fg} + x \frac{du_{fg}}{dp_{in}} \dot{p}_{in} + \frac{du_f}{dp_{in}} \dot{p}_{in} \right) + k h_g \left( p_{in} - p_{out} \right) \qquad (7)$$

It is possible to plug equation (3) into equation (7) to eliminate the quality, *x*. However, in the end, we are left with a non-linear differential equation that is difficult to make use of, except by solving it numerically.

## Simplified Analysis

Let's consider a more specific case which lends itself more readily to closed-formed analysis. Assume that the heat flow into the dewar, *q*, is constantly varying in such a way as to maintain the dewar pressure constant (equal to $p_{in}$). Combining (1) and (4), we get:

$$q = -\dot{m}_{out}\left( x u_{fg} + u_f \right) + M\left( \dot{x} u_{fg} + x \dot{u}_{fg} + \dot{u}_f \right) + \dot{m}_{out} h_g$$
$$= -\dot{m}_{out}\left( x u_{fg} + u_f \right) + M\left( \dot{x} u_{fg} + x \frac{du_{fg}}{dp_{in}} \dot{p}_{in} + \frac{du_f}{dp_{in}} \dot{p}_{in} \right) + \dot{m}_{out} h_g \qquad (8)$$

Realizing that $\dot{p}_{in}$ is now zero, this simplifies to:

$$\underbrace{q}_{\dot{E}_{in}} = \underbrace{-\dot{m}_{out}\left( x u_{fg} + u_f \right) + M \dot{x} u_{fg}}_{\frac{dE}{dt}} + \underbrace{\dot{m}_{out} h_g}_{\dot{E}_{out}} \qquad (9)$$

where we have noted the physical significance of the terms in the energy balance.
Now solving (3) for the quality:

$$x = \frac{1}{v_{fg}}\left( \frac{V}{M} - v_f \right), \qquad (10)$$

and differentiating in time:

$$\dot{x} = -\frac{\dot{M} V}{M^2 v_{fg}} = \frac{\dot{m}_{out} V}{M^2 v_{fg}}, \qquad (11)$$

Where again we have assumed the dewar pressure to be constant. Plugging (11) into (9), we obtain:

$$\begin{aligned} q &= -\dot{m}_{out}\left(xu_{fg}+u_f\right)+\dot{m}_{out}\frac{Vu_{fg}}{Mv_{fg}}+\dot{m}_{out}h_g \\ &= \dot{m}_{out}\left(-xu_{fg}-u_f+\frac{Vu_{fg}}{Mv_{fg}}+h_g\right) \end{aligned} \tag{12}$$

We can also plug in (10) to completely eliminate quality from the equation:

$$\begin{aligned} q &= \dot{m}_{out}\left[-\frac{1}{v_{fg}}\left(\frac{V}{M}-v_f\right)u_{fg}-u_f+\frac{Vu_{fg}}{Mv_{fg}}+h_g\right]=\dot{m}_{out}\left(\frac{v_f u_{fg}}{v_{fg}}-u_f+h_g\right) \\ &= \dot{m}_{out}\left(\frac{v_f u_{fg}}{v_{fg}}-\frac{v_{fg}u_f}{v_{fg}}+h_g\right)=\dot{m}_{out}\left[\frac{v_f\left(u_g-u_f\right)-\left(v_g-v_f\right)u_f}{v_{fg}}+h_g\right] \\ &= \dot{m}_{out}\left(\frac{v_f u_g-v_g u_f}{v_{fg}}+h_g\right)=\dot{m}_{out}\left(\frac{v_f u_g-v_g u_f}{v_g-v_f}+u_g+p_{in}v_g\right) \\ &= \dot{m}_{out}\left(\frac{v_f u_g-v_g u_f}{v_g-v_f}+\frac{u_g v_g-u_g v_f}{v_g-v_f}+p_{in}v_g\right)=\dot{m}_{out}\left(\frac{v_g\left(u_g-u_f\right)}{v_g-v_f}+p_{in}v_g\right) \\ &= \dot{m}_{out}\left(\frac{u_{fg}}{v_{fg}}v_g+p_{in}v_g\right) \end{aligned} \tag{13}$$

In the end, the amount of heat flow required to maintain a constant dewar pressure is:

$$\boxed{q\left(\dot{m}_{out},p_{in}\right)=\dot{m}_{out}v_g\left(\frac{u_{fg}}{v_{fg}}+p_{in}\right)} \tag{14}$$

where $q$ is shown as being a function of only $\dot{m}_{out}$ and $p_{in}$ because $v_g$, $v_{fg}$ and $u_{fg}$ are themselves functions of only $p_{in}$.

There are a few interesting things to note from this relation about the required heat flow, $q$:

1. As one might expect, it is simply proportional to $\dot{m}_{out}$.
2. It is independent of the size of the dewar (not as obvious).
3. It is independent of the amount of nitrogen in the dewar (not as obvious).

Figure 2 below shows a plot of $q/\dot{m}_{out}$ for nitrogen as a function of dewar pressure. It also shows how the saturation temperature (in the dewar) and the outlet temperature (at atmospheric pressure) vary with dewar pressure.

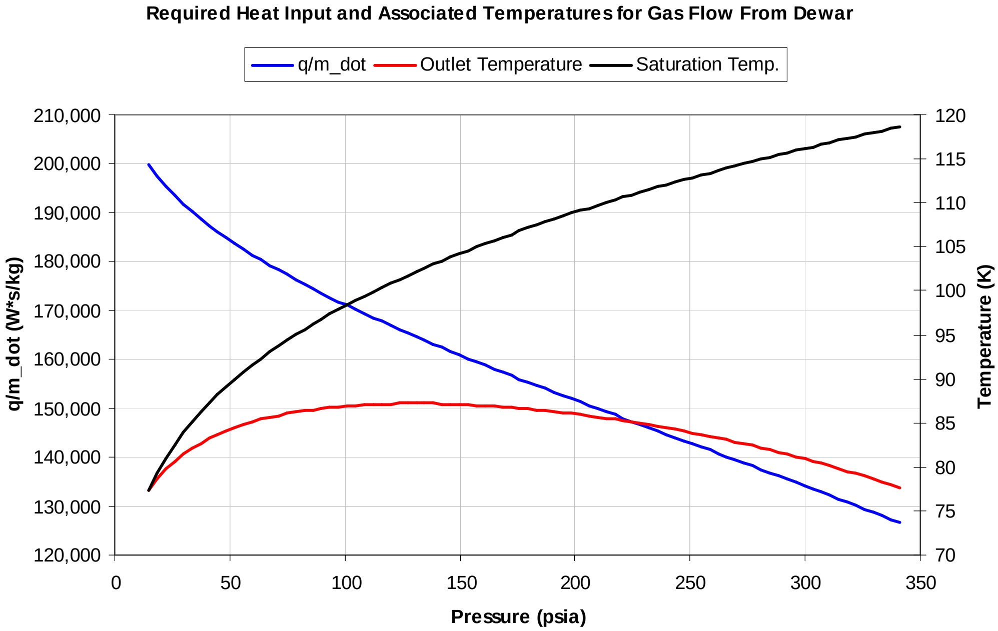


*Figure. 2. Plot of heat flow and temperatures for vapor flow from a $N_2$ dewar*

The following table shows a sample calculation:

| | | | | |
|---|---|---|---|---|
| p_in | 200000 | Pa | 29.02 | psia |
| | | | | |
| u_g (molar) | 1647.186 | J/mol | | |
| u_g (SI) | 58828.07 | J/kg | | |
| u_f (molar) | -3033.44 | J/mol | | |
| u_f (SI) | -108337 | J/kg | | |
| u_fg (molar) | 4680.626 | J/mol | | |
| u_fg (SI) | 167165.2 | J/kg | | |
| | | | | |
| rho_g (molar) | 0.30903 | mol/L | | |
| v_g (SI) | 0.115569 | m^3/kg | | |
| rho_f (molar) | 27.7945 | mol/L | | |
| v_f (SI) | 0.001285 | m^3/kg | | |
| v_fg (SI) | 0.114284 | m^3/kg | | |
| | | | | |
| h_g (molar) | 2294.38 | J/mol | | |
| h_g (SI) | 81942.14 | J/kg | | |
| | | | | |
| m_dot | 0.002 | kg/sec | 7.2 | kg/hr |
| | | | | |
| | | | | |
| **q_actual** | **384.32** | **Watts** | | |
| | | | | |
| q_est (m_dot*h_g) | 163.88 | Watts | | |

*Table 1. Example calculation for vapor flow from a $N_2$ dewar*

## *Liquid Flow*

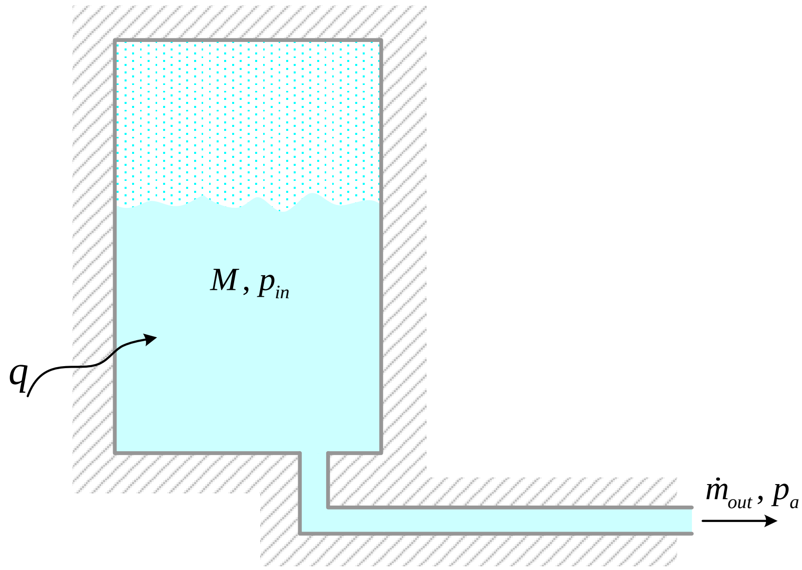


*Figure 3. Diagram of saturated liquid flow from a dewar*

Without much more work, we can look at a different scenario of flow from a dewar. In this case, it is saturated liquid that flows from the dewar instead of saturated vapor. To determine the required amount of heat to maintain the dewar pressure, we note the only difference is that the enthalpy of the exiting mass flow is now equal to the saturated liquid enthalpy, $h_f$, instead of the saturated vapor enthalpy, $h_g$. We can simply plug this into (13):

$$
\begin{aligned}
q &= \dot{m}_{out}\left(\frac{v_f u_g - v_g u_f}{v_{fg}} + h_f\right) = \dot{m}_{out}\left(\frac{v_f u_g - v_g u_f}{v_g - v_f} + u_f + p_{in} v_f\right) \\
&= \dot{m}_{out}\left(\frac{v_f u_g - v_g u_f}{v_g - v_f} + \frac{u_f v_g - u_f v_f}{v_g - v_f} + p_{in} v_f\right) = \dot{m}_{out}\left(\frac{v_f\left(u_g - u_f\right)}{v_g - v_f} + p_{in} v_f\right) \qquad (15) \\
&= \dot{m}_{out}\left(\frac{u_{fg}}{v_{fg}} v_f + p_{in} v_f\right)
\end{aligned}
$$

The final expression is:

$$\boxed{q\left(\dot{m}_{out}, p_{in}\right) = \dot{m}_{out} v_f \left(\frac{u_{fg}}{v_{fg}} + p_{in}\right)} \qquad (16)$$

## *Comparing saturated vapor and liquid flows*

Note how similar expression (16) is compared to that for saturated vapor flow in equation (14). In fact, the ratio of the two of them (for the same mass flow output) is:

$$\boxed{\frac{q_{vapor}}{q_{liquid}} = \frac{v_g}{v_f} = \frac{\rho_f}{\rho_g}} \qquad (17)$$

For nitrogen at 1 atmosphere, the ratio of saturated liquid to vapor densities is 175, implying it takes 175 times more heat flow (power) to maintain a saturated vapor flow than it takes to maintain the same mass flow rate of liquid.